Instructional Focus Document
Grade 8 Mathematics

TITLE : Unit 08: Angle and Triangle Relationships involving Real Numbers **SUGGESTED DURATION :** 13 days

UNIT OVERVIEW

Introduction

This unit bundles student expectations that address the real number system and relationships within triangles and angles including the Pythagorean Theorem, interior and exterior angles, transversals, and the angle-angle criterion for similarity of triangles. According to the Texas Education Agency, mathematical process standards including application, a problem-solving model, tools and techniques, communication, representations, relationships, and justifications should be integrated (when applicable) with content knowledge and skills so that students are prepared to use mathematics in everyday life, society, and the workplace. The introduction to the grade level standards state, “While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.” Additionally, the availability of graphing technology is required during STAAR testing.

Prior to this Unit

In Grade 6, students ordered a set of rational numbers arising from mathematical and real-world contexts. In Grade 7, students extended previous knowledge of sets and subsets using a visual representation to describe relationships between sets of rational numbers. They used models to determine the approximate formulas for the circumference and area of a circle and connected the models to the actual formulas. Students also wrote and solved equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.

During this Unit

Students extend previous knowledge of sets and subsets to order and describe relationships between sets of real numbers, which includes rational numbers and their subsets as well as irrational numbers. Students approximate the value of irrational numbers less than 225 and locate those approximations on a number line. Students are expected to use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. Right triangles are examined more closely within this unit as students use models to explain the Pythagorean Theorem. Students use the Pythagorean Theorem and its converse to solve problems and apply these understandings to the coordinate plane as they determine the distance between two points on the coordinate plane.

Other considerations: Reference the Mathematics COVID-19 Gap Implementation Tool Grade 8

After this Unit

In high school, students will extend concepts of determining the distance between two points on the coordinate plane using the Pythagorean Theorem to using the distance formula. They will investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools. Students will verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, midsegments, and medians, and apply these relationships to solve problems.

Additional Notes

In Grade 8, extending previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers and approximating the

value of an irrational number, including p and square roots of numbers less than 225, and locating that rational number approximation on a number line are identified as STAAR Supporting Standards 8.2A and 8.2B. Ordering a set of real numbers arising from mathematical and real-world contexts is STAAR Readiness Standard 8.2D. These standards are part of the Grade 8 *Texas Response to Curriculum Focal Points* (TxRCFP): Grade Level Connections (TxRCFP) and the Grade 8 STAAR Reporting Category 1: Numerical Representations and Relationships. Using models and diagrams to explain the Pythagorean theorem, determining the distance between two points on a coordinate plane using the Pythagorean Theorem, and using informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles are identified as STAAR Supporting Standards 8.6C, 8.7D, and 8.8D. Using the Pythagorean Theorem and its converse to solve problems is STAAR Readiness Standard 8.7C. These four standards are part of the Grade 8 Focal Point: Using expressions and equations to describe relationships, including the Pythagorean Theorem and the Grade 8 STAAR Reporting Category 3: Geometry and Measurement. This unit is supporting the development of the *Texas College and Career Readiness Standards* (TxCCRS): I. Numeric Reasoning A1, B1; II. Algebraic Reasoning A1, D1, D2; III. Geometric and Spatial Reasoning A2, A3, C1, D3; V. Statistical Reasoning A1, C2; VII. Problem Solving and Reasoning A1, A2, A3, A4, A5, B1, C1, D1, D2; VIII. Communication and Representation A1, A2, A3, B1, B2, C1, C2, C3; IX. Connections A1, A2, B1, B2, B3.

Research

According to research published by the National Mathematics Advisory Panel (2008), “The survey results reinforce the research findings presented in this report, particularly the need to strengthen students’ proficiency with rational numbers” (p.9). They further state, “Proficiency with whole numbers, fractions, and particular aspects of geometry and measurement should be understood as the Critical Foundations of Algebra. Emphasis on these essential concepts and skills must be provided at the elementary and middle grade levels” (p.18). The National Council of Teachers of Mathematics (2010) concludes, “Students who wait until Grade 9 to begin their high school mathematics coursework can benefit from productive time spent in grade 8 further developing their understanding of the rational number system…” (p 5). NCTM also notes that “As students work toward attaining these understandings (Pythagorean Theorem), they use both inductive and deductive reasoning to discover the properties related to distance and angle that are necessary to analyze geometric figures and space” (p. 52). “Students’ initial experiences with right triangles and the Pythagorean Theorem should be concrete” (NCTM, 2010, p. 67). In research documented by Van de Walle, Karp, and Bay-Williams (2010), “Students in the middle grades need to develop a more complete understanding of the number system, which includes extending whole numbers to integers and starting to think of fractions as rational numbers (both positive and negative). In these ways and others they can begin to appreciate the completeness of the real number system” (p. 473).

National Council of Teachers of Mathematics. (2010). *Focus in grade 8: Teaching with curriculum focal points*. Reston, VA: National Council of Teachers of Mathematics, Inc.
National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the national mathematics advisory panel.* Washington, DC: U.S. Department of Education.
Texas Education Agency & Texas Higher Education Coordinating Board. (2009). *Texas college and career readiness standards.* Retrieved from http://www.thecb.state.tx.us/institutional-resources-programs/public-community-technical-state-colleges/texas-college-and-career-readiness-standards/
Texas Education Agency. (2013). *Texas response to curriculum focal points for kindergarten through grade 8 mathematics.* Retrieved from https://www.texasgateway.org/resource/txrcfp-texas-response-curriculum-focal-points-k-8-mathematics-revised-2013
Van de Walle, J., Karp, K., & Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally*. Boston, MA: Pearson Education, Inc.

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OVERARCHING UNDERSTANDINGS AND QUESTIONS

Numeracy requires the ability to work flexibly with quantities in order to recognize, reason, and solve situations of varying contexts in everyday life, society, and the work place.

- How is numeracy like literacy?
- What are some examples of numeracy in everyday life, society, and the work place?
- How does context influence understanding of a quantity?
- Why is the ability to work flexibly with quantities essential to developing the foundations of numeracy?

UNIT UNDERSTANDINGS AND QUESTIONS	OVERARCHING CONCEPTS AND UNIT CONCEPTS	PERFORMANCE ASSESSMENT(S)

Real numbers create a more sophisticated number system where new relationships exist within and between sets and subsets of numbers *(real numbers)*.
• What representations can be used to visually demonstrate relationships between sets and subsets of numbers?
• How does organizing numbers in sets and subsets aid in understanding the relationships between real numbers?
• What relationships exist between sets and subsets of numbers?
• How are the elements in counting (natural) numbers,
	Number and Operations
	• Number
	  • Counting (Natural) numbers
	  • Whole numbers
	  • Integers
	  • Rational numbers
	  • Irrational numbers
	  • Real numbers
	  • Square roots
	• Number Representations
	  • Sets and subsets
	• Compare and Order
		Mathematics Grade 8 Unit 08 PA 01
		Click on the PA title to view related rubric.
		
		Analyze the situation(s) described below. Organize and record your work for each of the following tasks. Using precise mathematical language, justify and explain each mathematical process.
		
		1. Consider the numbers below.
		$-\sqrt{14.2}$ -4, 0.2, $\sqrt{2}$, $4\frac{1}{5}$, 0, -2.4, $\sqrt{6.25}$
		
		a. Create a visual representation to organize and display the relationship between the sets and

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UNIT UNDERSTANDINGS AND QUESTIONS	OVERARCHING CONCEPTS AND UNIT CONCEPTS	PERFORMANCE ASSESSMENT(S)
whole numbers, integers, rational numbers, irrational numbers, and real numbers related? • How can a number belong to the same set of numbers but not necessarily the same subset of numbers? • What relationship exists between … ◦ rational numbers and the other number sets? ◦ irrational numbers and the other number sets? ◦ real numbers and the other number sets? The ability to recognize and represent numbers in various forms develops the understanding of equivalence and allows for working flexibly with numbers in order to communicate and reason about the value of the number *(irrational numbers)*. • What is the process to approximate the value of an irrational number? • Why is it important to know how to algebraically approximate the value of an irrational number? • How does a calculator approximation of an irrational compare to a numeric approximation of an irrational number? • What is the relationship between the rational number approximations of $-\sqrt{x}$ and $\sqrt{x}$? Quantities are compared and ordered to determine magnitude of number and equality or inequality relations *(real numbers)*.	• Comparative language • Comparison symbols • Relationships and Generalizations • Numerical • Equivalence • Representations <u>Associated Mathematical Processes</u> • Tools and Techniques • Communication • Representations • Relationships • Justification	subsets of numbers: • counting (natural) numbers • integers • irrational numbers • rational numbers • real numbers • whole numbers b. Place the numbers in the correct set or subset within the visual representation. c. Record two additional numbers that belong in each set of counting (natural) numbers, integers, irrational numbers, and rational numbers within the visual representation. d. Approximate the value of each of the irrational numbers and locate those approximations on a number line. e. Place all of the numbers recorded within the visual representation in ascending order and verify the order using a calculator. Standard(s): **8.1C**, **8.1D**, **8.1E**, **8.1F**, **8.1G**, **8.2A**, **8.2B**, **8.2D**, **ELPS.c.1A**, **ELPS.c.2C**, **ELPS.c.2D**, **ELPS.c.2E**, **ELPS.c.3C**, **ELPS.c.3D**, **ELPS.c.3H**, **ELPS.c.4C**, **ELPS.c.4D**, **ELPS.c.4F**, **ELPS.c.4H**, **ELPS.c.5B**

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UNIT UNDERSTANDINGS AND QUESTIONS	OVERARCHING CONCEPTS AND UNIT CONCEPTS	PERFORMANCE ASSESSMENT(S)

• Why is it important to identify the unit or attribute being described by numbers before comparing or ordering the numbers?
• How can …
  ◦ place value
  ◦ numeric representations
  ◦ concrete representations
  ◦ pictorial representations
  … aid in the comparison and/or ordering of numbers?
• How can the comparison of two numbers be described and represented?
• How are quantifying descriptors used to determine the order of a set of numbers?		

OVERARCHING UNDERSTANDINGS AND QUESTIONS

Quantitative relationships model problem situations efficiently and can be used to make generalizations, predictions, and critical judgements in everyday life.
- What patterns exist within different types of quantitative relationships and where are they found in everyday life?
- Why is the ability to model quantitative relationships in a variety of ways essential to solving problems in everyday life?

UNIT UNDERSTANDINGS AND QUESTIONS	OVERARCHING CONCEPTS AND UNIT CONCEPTS	PERFORMANCE ASSESSMENT(S)
Illustrating and analyzing geometric relationships in	Expressions, Equations, and Relationships	Mathematics Grade 8 Unit 08 PA 02

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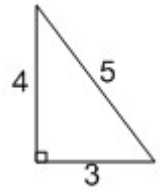
UNIT UNDERSTANDINGS AND QUESTIONS	OVERARCHING CONCEPTS AND UNIT CONCEPTS	PERFORMANCE ASSESSMENT(S)

models and diagrams aid in representing attributes of geometric figures with quantifiable measures and equations in order to generalize geometric relationships and solve problems.
• What models and diagrams represent and explain the Pythagorean Theorem?
• What is the relationship between the legs and hypotenuse in a right triangle?
• How can the Pythagorean Theorem be used to solve problems?
• What is the process to determine the length of a leg when the measure of the hypotenuse and other leg is given?
• How can the converse of the Pythagorean Theorem be used to determine if a triangle is a right triangle?
	• Composition and Decomposition of Figures and Angles
	• Geometric Representations
	  • One-dimensional representations
	  • Two-dimensional figures
	• Geometric Relationships
	  • Formulas
	  • Congruence
	  • Similarity
	  • Pythagorean Theorem
	  • Converse of Pythagorean Theorem
	  • Distance
	  • Measure relationships
	  • Geometric properties
	  • Informal arguments
		Click on the PA title to view related rubric.
		Provide students with square tiles, grid paper, and/or tangrams to select.
		Analyze the problem situation(s) described below. Organize and record your work for each of the following tasks. Using precise mathematical language, justify and explain each solution process.
		1. Consider the right triangle below.
		 

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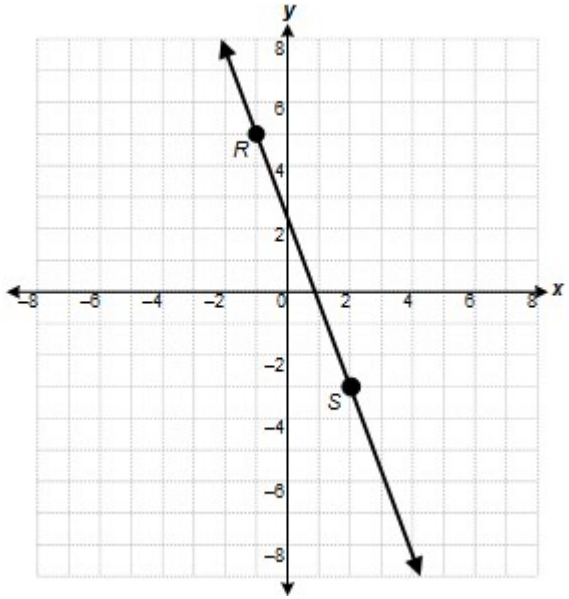
UNIT UNDERSTANDINGS AND QUESTIONS	OVERARCHING CONCEPTS AND UNIT CONCEPTS	PERFORMANCE ASSESSMENT(S)

• How can the Pythagorean Theorem be used to determine the distance between two points on a coordinate plane?
• What informal arguments can be made to establish facts about the …
  ◦ sum of the interior angles of a triangle?
  ◦ adjacent interior and exterior angles of a triangle?
  ◦ exterior angle of a triangle and the two non-adjacent interior angles or the remote interior angles?
  ◦ sum of the exterior angles of a triangle?
  ◦ angles created when two parallel lines are cut by a transversal?
  ◦ angle-angle criterion for similarity of triangles?
• What equation represents the relationship between the …
  ◦ sum of the interior angles of a triangle?
  ◦ adjacent interior and exterior angles of a triangle?
  ◦ exterior angle of a triangle and the two non-adjacent interior angles or the remote interior angles?
  ◦ sum of the exterior angles of a triangle?
• When two parallel lines are cut by a transversal, what equation represents the relationship between …
  ◦ alternate interior angles?
  ◦ alternate exterior angles?
  ◦ corresponding angles?
  ◦ adjacent angles?
	• Representations
	
	<u>Associated Mathematical Processes</u>
	• Application
	• Problem Solving Model
	• Tools and Techniques
	• Communication
	• Representations
	• Relationships
	• Justification
		a. Using models or diagrams, explain the Pythagorean Theorem.
		
		2. Consider the line below.
		 
		
		a. Determine the distance between points *R* and *S* on the coordinate plane using the Pythagorean Theorem.
		
		3. An architect has been commissioned to design an area for a rose garden. The rose garden will be between two parallel rows of trees and will be in the shape of a

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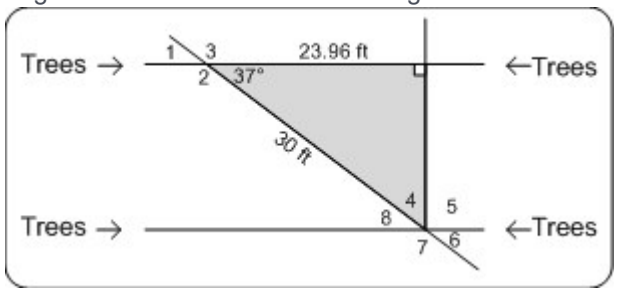
UNIT UNDERSTANDINGS AND QUESTIONS	OVERARCHING CONCEPTS AND UNIT CONCEPTS	PERFORMANCE ASSESSMENT(S)
◦ vertical angles? ◦ same side interior angles? ◦ same side exterior angles? • How can the angle-angle criterion be used to determine if two triangles are similar?		triangle as shown in the scale drawing below.  a. Determine the distance between the two parallel rows of trees in the garden. b. Using informal arguments to establish facts about the angles created when parallel lines are cut by a transversal, identify the measures of the seven unknown angles in the drawing. c. Using informal arguments to establish facts about the angle-angle criterion for similarity of triangles, describe how the scale drawing of the triangular rose garden and the actual triangular rose garden will be similar based on the measures of the interior angles of the two triangles. Standard(s): 8.1A, 8.1B, 8.1C, 8.1D, 8.1E, 8.1F, 8.1G, 8.6C, 8.7C, 8.7D, 8.8D, ELPS.c.1A, ELPS.c.2C, ELPS.c.2D, ELPS.c.2E, ELPS.c.3C, ELPS.c.3D, ELPS.c.3H, ELPS.c.3I, ELPS.c.4C, ELPS.c.4D,

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UNIT UNDERSTANDINGS AND QUESTIONS	OVERARCHING CONCEPTS AND UNIT CONCEPTS	PERFORMANCE ASSESSMENT(S)
		ELPS.c.4F, ELPS.c.4H, ELPS.c.4J, ELPS.c.5B, ELPS.c.5F, ELPS.c.5G

MISCONCEPTIONS / UNDERDEVELOPED CONCEPTS

Misconceptions:

- Some students may mislabel the hypotenuse as *a* or *b* rather than labeling it as *c*.
- Some students may think the Pythagorean relationship can be used on all triangles instead of only right triangles.

Underdeveloped Concepts:

- Some students may think they should take half of a number when calculating the square root of the number.
- Some students may think they should double a number when squaring the number.

UNIT VOCABULARY

- **Adjacent angles** – two non-overlapping angles that share a common vertex and exactly one ray
- **Angle** – two rays with a common end point (the vertex)
- **Angle-angle criterion for triangles** – if two angles in one triangle are congruent to two angles in another triangle, then the measures of the third angle in both triangles are congruent
- **Axes** – the vertical and horizontal lines that act as a reference when plotting points on a coordinate plane
- **Complementary angles** – two angles whose degree measures have a sum of 90°
- **Congruent angles** – angles whose angle measurements are equal
- **Coordinate plane (coordinate grid)** – a two-dimensional plane on which to plot points, lines, and curves
- **Counting (natural) numbers** – the set of positive numbers that begins at one and increases by increments of one each time {1, 2, 3, …, *n*}

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- **Degree** – the measure of an angle where each degree represents $\frac{1}{360}$ of a circle
- **Exterior angles of a triangle** – angles that are outside of a triangle between one side of a triangle and the extension of the adjacent side
- **Hypotenuse** – the longest side of a right triangle, the side opposite the right angle
- **Integers** – the set of counting (natural numbers), their opposites, and zero {–*n*, …, –3, –2, –1, 0, 1, 2, 3, ..., *n*}. The set of integers is denoted by the symbol Z.
- **Interior angles of a triangle** – angles that are inside of a triangle, formed by two sides of the triangle
- **Intersecting lines** – lines that meet or cross at a point
- **Irrational numbers** – the set of numbers that cannot be expressed as a fraction $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$
- **Legs of a right triangle** – the two shortest sides of a right triangle
- **Order numbers** – to arrange a set of numbers based on their numerical value
- **Origin** – the starting point in locating points on a coordinate plane
- **Parallel lines** – lines that lie in the same plane, never intersect, and are always the same distance apart
- **Place value** – the value of a digit as determined by its location in a number such as ones, tens, hundreds, one thousands, ten thousands, etc.
- **Quadrants** – any of the four areas created by dividing a plane with an *x*-axis and *y*-axis
- **Rational numbers** – the set of numbers that can be expressed as a fraction $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$. The set of rational numbers is denoted by the symbol Q.
- **Real numbers** – the set of rational and irrational numbers. The set of real numbers is denoted by the symbol $\mathfrak{R}$.
- **Right triangle** – a triangle with one right angle (exactly 90°) and two acute angles
- **Square root** – a factor of a number that, when squared, equals the original number
- **Supplementary angles** – two angles whose degree measures have a sum of 180°
- **Transversal** – a line that intersects two or more lines
- **Triangle** – a polygon with three sides and three vertices
- **Vertical angles** – a pair of non-adjacent, non-overlapping angles formed by two intersecting lines creating angles that are opposite and congruent to each other
- **Whole numbers** – the set of counting (natural) numbers and zero {0, 1, 2, 3, ..., *n*}

Related Vocabulary:

- Ascending
- Consecutive
- Converse
- Corresponding angles
- Interval
- Number line
- Open number line
- Ordered pair
- Repeating decimal
- Square
- Terminating decimal
- *x*-axis

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- Descending
- Formula
- Informal argument
- Perfect square
- Pythagorean theorem
- Radical symbol
- *x* coordinate
- *y*-axis
- *y* coordinate

UNIT ASSESSMENT ITEMS	SYSTEM RESOURCES	OTHER RESOURCES
Unit Assessment Items that have been published by your district may be accessed through **Search All Components** in the District Resources tab. Assessment items may also be found using the Assessment Center if your district has granted access to that tool.	**Mathematics Concepts Charts** **Mathematics COVID-19 Gap Implementation Tool Grade 8** **Mathematics COVID-19 Gap Implementation Tool Instructions** **Mathematics Grade 8 Backward Design Document** **Mathematics Grade 8 Enhanced TEKS Clarification** **Mathematics Grade 8 Focal Points with Aligned Standards and TEKS Introduction** **Mathematics Grade 8 STAAR Analysis Resources** **Mathematics Grade 8 STAAR Blueprint and Item Percentages** **Mathematics Grade 8 STAAR Enhanced Blueprint** **Mathematics Grade 8 Vertical Alignment** **Mathematics Grade 8_Unit 08_TEKS Resource**	Texas Higher Education Coordinating Board – **Texas College and Career Readiness Standards** Texas Education Agency – **Texas Response to Curriculum Focal Points for K-8 Mathematics Revised 2013** Texas Education Agency – **Mathematics Curriculum** Texas Education Agency – **STAAR Mathematics Resources** Texas Education Agency Texas Gateway – **Revised Mathematics TEKS: Vertical Alignment Charts** Texas Education Agency Texas Gateway – **Mathematics TEKS: Supporting Information** Texas Education Agency Texas Gateway – **Interactive Mathematics Glossary** Texas Education Agency Texas Gateway – **Resources Aligned to Grade 8 Mathematics TEKS** Texas Instruments – **Graphing Calculator Tutorials**

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	System STAAR Analysis **Mathematics K-HS Overarching Understandings and Questions** **Mathematics Long Term Transfer Goals** **Mathematics Suggested Basic Manipulatives by Grade Level** **Mathematics Suggested Engaging Literature** **Mathematics Texas Education Agency Grade 8 TEKS Supporting Information (with TEKS Resource System Comments)** **Mathematics Vertical Quick Guide**	

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TAUGHT DIRECTLY TEKS

TEKS INTENDED TO BE EXPLICITLY TAUGHT IN THIS UNIT.

TEKS/SE Legend:

- ***Knowledge and Skills Statements (TEKS) identified by TEA are in italicized, bolded, black text.***
- **Student Expectations (TEKS) identified by TEA are in bolded, black text.**
- ***Student Expectations (TEKS) are labeled Readiness as identified by TEA of the assessed curriculum.***
- ***Student Expectations (TEKS) are labeled Supporting as identified by TEA of the assessed curriculum.***
- ***Student Expectations (TEKS) are labeled Process standards as identified by TEA of the assessed curriculum.***
- Portions of the Student Expectations (TEKS) that are not included in this unit but are taught in previous or future units are indicated by a ~~strike-through~~.

Specificity Legend:

- Supporting information / clarifications (specificity) written by TEKS Resource System are in blue text.
- *Unit-specific clarifications are in italicized, blue text.*
- Information from Texas Education Agency (TEA), Texas College and Career Readiness Standards (TxCCRS), Texas Response to Curriculum Focal Points (TxRCFP) is labeled.

TEKS# / SE#	TEKS	SPECIFICITY
8.1	***Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:***	
8.1A	**Apply mathematics to problems arising in everyday life, society, and the workplace.** ***Process Standard***	Apply MATHEMATICS TO PROBLEMS ARISING IN EVERYDAY LIFE, SOCIETY, AND THE WORKPLACE

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TEKS# / SE#	TEKS	SPECIFICITY

		Including, but not limited to:
		• Mathematical problem situations within and between disciplines
		  ◦ Everyday life
		  ◦ Society
		  ◦ Workplace
		
		Note(s):
		• The mathematical process standards may be applied to all content standards as appropriate.
		• TxRCFP:
		  ◦ Representing, applying, and analyzing proportional relationships
		  ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem
		  ◦ Making inferences from data
		• TxCCRS:
		  ◦ VII.D. Problem Solving and Reasoning – Real-world problem solving
		  • VII.D.1. Interpret results of the mathematical problem in terms of the original real-world situation.
		  ◦ IX.A. Connections – Connections among the strands of mathematics
		  • IX.A.1. Connect and use multiple key concepts of mathematics in situations and problems.
		  • IX.A.2. Connect mathematics to the study of other disciplines.
		  ◦ IX.B. Connections – Connections of mathematics to nature, real-world situations, and everyday life
		  • IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
		  • IX.B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.

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TEKS# / SE#	TEKS	SPECIFICITY
		• IX.B.3. Know and understand the use of mathematics in a variety of careers and professions.

8.1B	**Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.** ***Process Standard***	Use
		
		A PROBLEM-SOLVING MODEL THAT INCORPORATES ANALYZING GIVEN INFORMATION, FORMULATING A PLAN OR STRATEGY, DETERMINING A SOLUTION, JUSTIFYING THE SOLUTION, AND EVALUATING THE PROBLEM-SOLVING PROCESS AND THE REASONABLENESS OF THE SOLUTION
		
		Including, but not limited to:
		
		• Problem-solving model
		  ◦ Analyze given information
		  ◦ Formulate a plan or strategy
		  ◦ Determine a solution
		  ◦ Justify the solution
		  ◦ Evaluate the problem-solving process and the reasonableness of the solution
		
		Note(s):
		
		• The mathematical process standards may be applied to all content standards as appropriate.
		• TxRCFP:
		  ◦ Representing, applying, and analyzing proportional relationships
		  ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem
		  ◦ Making inferences from data
		• TxCCRS:
		  ◦ I.B. Numeric Reasoning – Number sense and number concepts

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TEKS# / SE#	TEKS	SPECIFICITY
		• I.B.1. Use estimation to check for errors and reasonableness of solutions. ◦ V.A. Statistical Reasoning – Design a study • V.A.1. Formulate a statistical question, plan an investigation, and collect data. ◦ VII.A. Problem Solving and Reasoning – Mathematical problem solving • VII.A.1. Analyze given information. • VII.A.2. Formulate a plan or strategy. • VII.A.3. Determine a solution. • VII.A.4. Justify the solution. • VII.A.5. Evaluate the problem-solving process. ◦ VII.D. Problem Solving and Reasoning – Real-world problem solving • VII.D.2. Evaluate the problem-solving process.
8.1C	**Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.** ***Process Standard***	Select TOOLS, INCLUDING REAL OBJECTS, MANIPULATIVES, PAPER AND PENCIL, AND TECHNOLOGY AS APPROPRIATE, AND TECHNIQUES, INCLUDING MENTAL MATH, ESTIMATION, AND NUMBER SENSE AS APPROPRIATE, TO SOLVE PROBLEMS Including, but not limited to: • Appropriate selection of tool(s) and techniques to apply in order to solve problems ◦ Tools • Real objects • Manipulatives • Paper and pencil • Technology ◦ Techniques • Mental math • Estimation

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		• Number sense
		
		Note(s):
		
		• The mathematical process standards may be applied to all content standards as appropriate.
		• TxRCFP:
		  ◦ Representing, applying, and analyzing proportional relationships
		  ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem
		  ◦ Making inferences from data
		• TxCCRS:
		  ◦ I.B. Numeric Reasoning – Number sense and number concepts
		  • I.B.1. Use estimation to check for errors and reasonableness of solutions.
		  ◦ V.C. Statistical Reasoning – Analyze, interpret, and draw conclusions from data
		  • V.C.2. Analyze relationships between paired data using spreadsheets, graphing calculators, or statistical software.

8.1D	**Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.**
	Process Standard
		Communicate
		
		MATHEMATICAL IDEAS, REASONING, AND THEIR IMPLICATIONS USING MULTIPLE REPRESENTATIONS, INCLUDING SYMBOLS, DIAGRAMS, GRAPHS, AND LANGUAGE AS APPROPRIATE
		
		Including, but not limited to:
		
		• Mathematical ideas, reasoning, and their implications
		  ◦ Multiple representations, as appropriate
		  • Symbols
		  • Diagrams

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		• Graphs
		• Language
		
		Note(s):
		
		• The mathematical process standards may be applied to all content standards as appropriate.
		• TxRCFP:
		  ◦ Representing, applying, and analyzing proportional relationships
		  ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem
		  ◦ Making inferences from data
		• TxCCRS:
		  ◦ II.D. Algebraic Reasoning – Representing relationships
		  • II.D.1. Interpret multiple representations of equations, inequalities, and relationships.
		  • II.D.2. Convert among multiple representations of equations, inequalities, and relationships.
		  ◦ VIII.A. Communication and Representation – Language, terms, and symbols of mathematics
		  • VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
		  • VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
		  • VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
		  ◦ VIII.B. Communication and Representation – Interpretation of mathematical work
		  • VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
		  • VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.

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		◦ VIII.C. Communication and Representation – Presentation and representation of mathematical work • VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words. • VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas. • VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications. ◦ IX.B. Connections – Connections of mathematics to nature, real-world situations, and everyday life • IX.B.1. Use multiple representations to demonstrate links between mathematical and real-world situations.
8.1E	**Create and use representations to organize, record, and communicate mathematical ideas.** ***Process Standard***	Create, Use REPRESENTATIONS TO ORGANIZE, RECORD, AND COMMUNICATE MATHEMATICAL IDEAS Including, but not limited to: • Representations of mathematical ideas ◦ Organize ◦ Record ◦ Communicate • Evaluation of the effectiveness of representations to ensure clarity of mathematical ideas being communicated • Appropriate mathematical vocabulary and phrasing when communicating mathematical ideas Note(s):

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		• The mathematical process standards may be applied to all content standards as appropriate.
		• TxRCFP:
		  ◦ Representing, applying, and analyzing proportional relationships
		  ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem
		  ◦ Making inferences from data
		• TxCCRS:
		  ◦ VIII.B. Communication and Representation – Interpretation of mathematical work
		  • VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
		  • VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
		  ◦ VIII.C. Communication and Representation – Presentation and representation of mathematical work
		  • VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
		  • VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.

8.1F	**Analyze mathematical relationships to connect and communicate mathematical ideas.**
	Process Standard
		Analyze
		
		MATHEMATICAL RELATIONSHIPS TO CONNECT AND COMMUNICATE MATHEMATICAL IDEAS
		
		Including, but not limited to:
		
		• Mathematical relationships
		  ◦ Connect and communicate mathematical ideas
		  • Conjectures and generalizations from sets of examples and non-examples, patterns, etc.

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		• Current knowledge to new learning
		
		Note(s):
		
		• The mathematical process standards may be applied to all content standards as appropriate.
		• TxRCFP:
		  ◦ Representing, applying, and analyzing proportional relationships
		  ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem
		  ◦ Making inferences from data
		• TxCCRS:
		  ◦ VII.A. Problem Solving and Reasoning – Mathematical problem solving
		  • VII.A.1. Analyze given information.
		  ◦ VIII.A. Communication and Representation – Language, terms, and symbols of mathematics
		  • VIII.A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
		  • VIII.A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
		  • VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing.
		  ◦ VIII.B. Communication and Representation – Interpretation of mathematical work
		  • VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations.
		  ◦ VIII.C. Communication and Representation – Presentation and representation of mathematical work
		  • VIII.C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, models, graphs, and words.
		  • VIII.C.2. Create and use representations to organize, record, and communicate mathematical ideas.

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		• VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications. ◦ IX.A. Connections – Connections among the strands of mathematics • IX.A.1. Connect and use multiple key concepts of mathematics in situations and problems. • IX.A.2. Connect mathematics to the study of other disciplines.
8.1G	**Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.** ***Process Standard***	Display, Explain, Justify MATHEMATICAL IDEAS AND ARGUMENTS USING PRECISE MATHEMATICAL LANGUAGE IN WRITTEN OR ORAL COMMUNICATION Including, but not limited to: • Mathematical ideas and arguments ◦ Validation of conclusions • Displays to make work visible to others ◦ Diagrams, visual aids, written work, etc. • Explanations and justifications ◦ Precise mathematical language in written or oral communication Note(s): • The mathematical process standards may be applied to all content standards as appropriate. • TxRCFP: ◦ Representing, applying, and analyzing proportional relationships ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem ◦ Making inferences from data

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		• TxCCRS: ◦ VII.A. Problem Solving and Reasoning – Mathematical problem solving • VII.A.4. Justify the solution. ◦ VII.B. Problem Solving and Reasoning – Proportional reasoning • VII.B.1. Use proportional reasoning to solve problems that require fractions, ratios, percentages, decimals, and proportions in a variety of contexts using multiple representations. ◦ VII.C. Problem Solving and Reasoning – Logical reasoning • VII.C.1. Develop and evaluate convincing arguments. ◦ VIII.A. Communication and Representation – Language, terms, and symbols of mathematics • VIII.A.3. Use mathematical language for reasoning, problem solving, making connections, and generalizing. ◦ VIII.B. Communication and Representation – Interpretation of mathematical work • VIII.B.1. Model and interpret mathematical ideas and concepts using multiple representations. • VIII.B.2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context. ◦ VIII.C. Communication and Representation – Presentation and representation of mathematical work • VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.
8.2	***Number and operations. The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:***	

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8.2A	**Extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers.** ***Supporting Standard***	Extend PREVIOUS KNOWLEDGE OF SETS AND SUBSETS USING A VISUAL REPRESENTATION Including, but not limited to: • Counting (natural) numbers – the set of positive numbers that begins at one and increases by increments of one each time {1, 2, 3, ..., *n*} • Whole numbers – the set of counting (natural) numbers and zero {0, 1, 2, 3, ..., *n*} • Integers – the set of counting (natural numbers), their opposites, and zero {–*n*, …, –3, –2, –1, 0, 1, 2, 3, ..., *n*}. The set of integers is denoted by the symbol Z. • Rational numbers – the set of numbers that can be expressed as a fraction $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$. The set of rational numbers is denoted by the symbol Q. • Irrational numbers – the set of numbers that cannot be expressed as a fraction $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$ • Real numbers – the set of rational and irrational numbers. The set of real numbers is denoted by the symbol ℜ. • Visual representations of the relationships between sets and subsets of real numbers To Describe RELATIONSHIPS BETWEEN SETS OF REAL NUMBERS Including, but not limited to: • All counting (natural) numbers are a subset of whole numbers, integers, rational numbers, and real numbers. • All whole numbers are a subset of integers, rational numbers, and real numbers.

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		• All integers are a subset of rational numbers and real numbers. • All counting (natural) numbers, whole numbers, and integers are a subset of rational numbers and real numbers. • Not all rational numbers are integers, whole numbers, or counting (natural) numbers. • Terminating and repeating decimals are rational numbers but not integers, whole numbers, or counting (natural) numbers. • All irrational numbers are a subset of real numbers. • Real numbers include all rational numbers, integers, whole numbers, counting (natural) numbers, and irrational numbers. • Not all real numbers are rational numbers. Note(s): • Grade Level(s): ◦ Grade 5 classified two-dimensional figures in a hierarchy of sets and subsets using graphic organizers based on their attributes and properties. ◦ Grade 6 classified whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers. ◦ Grade 7 extended previous knowledge of sets and subsets using a visual representation to describe relationships between sets of rational numbers. ◦ Grade 8 introduces the set of irrational numbers as a subset of real numbers. ◦ Various mathematical process standards will be applied to this student expectation as appropriate. • TxRCFP: ◦ Grade Level Connections (reinforces previous learning and/or provides development for future learning)
8.2B	**Approximate the value of an irrational number, including π and square roots of numbers less than**	Approximate

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	225, and locate that rational number approximation on a number line. ***Supporting Standard***	THE VALUE OF AN IRRATIONAL NUMBER, INCLUDING π AND SQUARE ROOTS OF NUMBERS LESS THAN 225 Including, but not limited to: • Irrational numbers – the set of numbers that cannot be expressed as a fraction $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$ ◦ Rational number approximations of irrational numbers to the appropriate place value for context of mathematical and real-world problem situations ◦ Approximation symbol (≈) • Square root – a factor of a number that, when squared, equals the original number ◦ Radical symbol ($\sqrt{x}$) • $\sqrt{x}$ represents the principal square root of *x*, the positive square root • $-\sqrt{x}$ represents the opposite of the principal square root of *x*, the negative square root • Rational number approximations of square roots less than 225 ◦ Integers ◦ Decimals ◦ Fractions • Verification of rational number approximations of irrational numbers with a calculator • Relationship between rational number approximations of perfect squares and irrational numbers ◦ Perfect squares of consecutive integers Locate RATIONAL NUMBER APPROXIMATIONS OF IRRATIONAL NUMBERS ON A NUMBER LINE Including, but not limited to:

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		• Rational numbers – the set of numbers that can be expressed as a fraction $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$. The set of rational numbers is denoted by the symbol Q.
		• Irrational numbers – the set of numbers that cannot be expressed as a fraction $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$
		• All rational number approximations of irrational numbers can be located on a number line.
		  ◦ Characteristics of a number line
		  • A number line begins as a line with predetermined intervals (or tick marks) with positions/numbers labeled.
		  ◦ A minimum of two positions/numbers should be labeled.
		  • Numbers on a number line represent the distance from zero.
		  • The distance between the tick marks is counted rather than the tick marks themselves.
		  • The placement of the labeled positions/numbers on a number line determines the scale of the number line.
		  ◦ Intervals between position/numbers are proportional.
		  • When reasoning on a number line, the position of zero may or may not be placed.
		  • When working with larger numbers, a number line without the constraint of distance from zero allows the ability to “zoom-in” on the relevant section of the number line.
		  • Number lines extend infinitely in both directions (arrows indicate the number line continues infinitely).
		  • Numbers increase from left to right on a horizontal number line and from bottom to top on a vertical number line.
		  ◦ Points to the left of a specified point on a horizontal number line are less than points to the right.
		  ◦ Points to the right of a specified point on a horizontal number line are greater than points to the left.
		  ◦ Points below a specified point on a vertical number line are less than

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		points above. ◦ Points above a specified point on a vertical number line are greater than points below. ◦ Characteristics of an open number line • An open number line begins as a line with no intervals (or tick marks) and no positions/numbers labeled. • Numbers/positions are placed on the empty number line only as they are needed. • When reasoning on an open number line, the position of zero is often not placed. ◦ When working with larger numbers, an open number line without the constraint of distance from zero allows the ability to “zoom-in” on the relevant section of the number line. • The placement of the first two numbers on an open number line determines the scale of the number line. ◦ Once the scale of the number line has been established by the placement of the first two numbers, intervals between additional numbers placed are approximately proportional. • The differences between numbers are approximated by the distance between the positions on the number line. • Open number lines extend infinitely in both directions (arrows indicate the number line continues infinitely). • Numbers increase from left to right on a horizontal number line and from bottom to top on a vertical number line. ◦ Points to the left of a specified point on a horizontal number line are less than points to the right. ◦ Points to the right of a specified point on a horizontal number line are greater than points to the left. • Landmark (or anchor) numbers may be placed on the open number line to help locate other numbers. • Rational number approximations of irrational numbers

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		• Rational number approximations of square roots less than 225
		  ◦ Integers
		  ◦ Decimals
		  ◦ Fractions
		• Verification of rational number approximations of irrational numbers with a calculator
		• Relationship between rational number approximations of perfect squares and irrational numbers
		  ◦ Perfect squares of consecutive integers
		
		Note(s):
		
		• Grade Level(s):
		  ◦ Grade 8 introduces approximating the value of an irrational number, including π and square roots of numbers less than 225, and locating that rational number approximation on a number line.
		  ◦ Various mathematical process standards will be applied to this student expectation as appropriate.
		• TxRCFP:
		  ◦ Grade Level Connections (reinforces previous learning and/or provides development for future learning)
		• TxCCRS:
		  ◦ I.B. Numeric Reasoning – Number sense and number concepts
		  • I.B.1. Use estimation to check for errors and reasonableness of solutions.

8.2D	**Order a set of real numbers arising from mathematical and real-world contexts.** ***Readiness Standard***	Order A SET OF REAL NUMBERS ARISING FROM MATHEMATICAL AND REAL-WORLD CONTEXTS Including, but not limited to:

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		• Real numbers – the set of rational and irrational numbers. The set of real numbers is denoted by the symbol ℜ.
		• Various forms of real numbers
		  ◦ Rational numbers (positive or negative)
		  ◦ Irrational numbers (positive or negative)
		• Place value – the value of a digit as determined by its location in a number such as ones, tens, hundreds, one thousands, ten thousands, etc.
		• Order numbers – to arrange a set of numbers based on their numerical value
		• Number lines (horizontal/vertical)
		• Numbers increase from left to right on a horizontal number line and from bottom to top on a vertical number line.
		  ◦ Points to the left of a specified point on a horizontal number line are less than points to the right.
		  ◦ Points to the right of a specified point on a horizontal number line are greater than points to the left.
		  ◦ Points below a specified point on a vertical number line are less than points above.
		  ◦ Points above a specified point on a vertical number line are greater than points below.
		• Quantifying descriptor in mathematical and real-world problem situations (e.g., between two given numbers, greatest/least, ascending/descending, tallest/shortest, warmest/coldest, fastest/slowest, longest/shortest, heaviest/lightest, closest/farthest, oldest/youngest, etc.)
		
		Note(s):
		
		• Grade Level(s):
		  ◦ Grade 6 ordered a set of rational numbers arising from mathematical and real-world contexts.
		  ◦ Grade 8 introduces ordering a set of real numbers arising from mathematical and real-world contexts.
		  ◦ Various mathematical process standards will be applied to this student expectation as appropriate.

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		• TxRCFP:
		  ◦ Grade Level Connections (reinforces previous learning and/or provides development for future learning)
		• TxCCRS:
		  ◦ I. Numeric Reasoning
		  ◦ IX. Communication and Representation
		  ◦ X. Connections

8.6	***Expressions, equations, and relationships. The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas. The student is expected to:***	

8.6C	**Use models and diagrams to explain the Pythagorean theorem.**
	Supporting Standard
		Use
		
		MODELS AND DIAGRAMS TO EXPLAIN THE PYTHAGOREAN THEOREM
		
		Including, but not limited to:
		
		• Right triangle – a triangle with one right angle (exactly 90°) and two acute angles
		• Legs of a right triangle – the two shortest sides of a right triangle
		• Hypotenuse – the longest side of a right triangle, the side opposite the right angle
		• Pythagorean theorem
		  ◦ Verbal: sum of the squares of the legs equals the square of the hypotenuse
		  ◦ Formula: $a^2 + b^2 = c^2$, where a and b represent the legs and c represents the hypotenuse
		• Models and diagrams
		  ◦ Square tiles

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		◦ Grid paper ◦ Tangrams Note(s): • Grade Level(s): ◦ Various mathematical process standards will be applied to this student expectation as appropriate. • TxRCFP: ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem • TxCCRS: ◦ II.D. Algebraic Reasoning – Representing relationships • II.D.1. Interpret multiple representations of equations, inequalities, and relationships. ◦ III.A. Geometric and Spatial Reasoning – Figures and their properties • III.A.3. Recognize and apply right triangle relationships including basic trigonometry. ◦ III.D. Geometric and Spatial Reasoning – Measurements involving geometry and algebra • III.D.3. Determine indirect measurements of geometric figures using a variety of methods.
8.7	***Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:***	
8.7C	**Use the Pythagorean Theorem and its converse to solve problems.**	Use

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	Readiness Standard	THE PYTHAGOREAN THEOREM AND ITS CONVERSE TO SOLVE PROBLEMS
		
		Including, but not limited to:
		
		• Right triangle – a triangle with one right angle (exactly 90°) and two acute angles
		• Legs of a right triangle – the two shortest sides of a right triangle
		• Hypotenuse – the longest side of a right triangle, the side opposite the right angle
		• Pythagorean Theorem
		  ◦ Verbal
		  • The sum of the squares of the legs of a right triangle equals the square of the hypotenuse.
		  ◦ Formula
		  • $a^2 + b^2 = c^2$, where a and b represent the legs of a right triangle and c represents the hypotenuse
		  • When solving for a, b, or c both the positive and negative numerical values should be considered, but since the applications are measurements the negative values do not apply.
		• Converse of Pythagorean Theorem
		  ◦ Verbal
		  • If the sum of the squares of the two shortest sides of a triangle equals the square of the third side, then the triangle is a right triangle.
		  ◦ Formula
		  • $a^2 + b^2 = c^2$, where a and b represent the legs of a right triangle and c represents the hypotenuse
		
		Note(s):
		
		• Grade Level(s):
		  ◦ Grade 8 introduces using the Pythagorean Theorem and its converse to solve problems.

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		◦ Various mathematical process standards will be applied to this student expectation as appropriate. • TxRCFP: ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem • TxCCRS: ◦ II.A. Algebraic Reasoning – Identifying expressions and equations • II.A.1. Explain the difference between expressions and equations. ◦ III.A. Geometric and Spatial Reasoning – Figures and their properties • III.A.3. Recognize and apply right triangle relationships including basic trigonometry. ◦ III.D. Geometric and Spatial Reasoning – Measurements involving geometry and algebra • III.D.3. Determine indirect measurements of geometric figures using a variety of methods.
8.7D	**Determine the distance between two points on a coordinate plane using the Pythagorean Theorem.** ***Supporting Standard***	Determine THE DISTANCE BETWEEN TWO POINTS ON A COORDINATE PLANE USING THE PYTHAGOREAN THEOREM Including, but not limited to: • Coordinate plane (coordinate grid) – a two-dimensional plane on which to plot points, lines, and curves • Axes – the vertical and horizontal lines that act as a reference when plotting points on a coordinate plane • Intersecting lines – lines that meet or cross at a point • Origin – the starting point in locating points on a coordinate plane • Quadrants – any of the four areas created by dividing a plane with an *x*-axis and *y*-axis

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		• Attributes of the coordinate plane
		  ◦ Two number lines intersect perpendicularly to form the axes, which are used to locate points on the plane.
		  • The horizontal number line is called the *x*-axis.
		  • The vertical number line is called the *y*-axis.
		  ◦ The *x*-axis and the *y*-axis cross at 0 on both number lines and that intersection is called the origin.
		  • The ordered pair of numbers corresponding to the origin is (0, 0)
		  ◦ Four quadrants are formed by the intersection of the *x*-and *y*-axes and are labeled counterclockwise with Roman numerals.
		  ◦ Iterated units are labeled and shown on both axes to show scale.
		  • Intervals may or may not be increments of one.
		  • Intervals may or may not include decimal or fractional amounts.
		  ◦ Relationship between ordered pairs and attributes of the coordinate plane
		  • A pair of ordered numbers names the location of a point on a coordinate plane.
		  • Ordered pairs of numbers are indicated within parentheses and separated by a comma. (*x*, *y*).
		  ◦ The first number in the ordered pair represents the parallel movement on the *x*-axis, left or right starting at the origin.
		  ◦ The second number in the ordered pair represents the parallel movement on the *y*-axis, up or down starting at the origin.
		• Right triangle – a triangle with one right angle (exactly 90°) and two acute angles
		• Legs of a right triangle – the two shortest sides of a right triangle
		• Hypotenuse – the longest side of a right triangle, the side opposite the right angle
		• Pythagorean Theorem
		  ◦ Verbal
		  • The sum of the squares of the legs of a right triangle equals the square of the hypotenuse.
		  ◦ Formula

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		• $a^2 + b^2 = c^2$, where a and b represent the legs of a right triangle and c represents the hypotenuse • When solving for a, b, or c both the positive and negative numerical values should be considered, but since the applications are measurements the negative values do not apply. • Generalizations from points on a coordinate plane ◦ A right triangle can be formed from any two points on a non-horizontal, non-vertical line by drawing a vertical line from one point and a horizontal line from the other point until the lines intersect. ◦ The Pythagorean Theorem can be used to determine the distance between two points on a coordinate plane. Note(s): • Grade Level(s): ◦ Grade 8 introduces determining the distance between two points on a coordinate plane using the Pythagorean Theorem. ◦ Geometry will derive and use the distance formula to verify geometric relationships and solve problems. ◦ Various mathematical process standards will be applied to this student expectation as appropriate. • TxRCFP: ◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem • TxCCRS: ◦ II.A. Algebraic Reasoning – Identifying expressions and equations • II.A.1. Explain the difference between expressions and equations. ◦ III.A. Geometric and Spatial Reasoning – Figures and their properties • III.A.3. Recognize and apply right triangle relationships including basic trigonometry.

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TEKS# / SE#	TEKS	SPECIFICITY
		◦ III.D. Geometric and Spatial Reasoning – Measurements involving geometry and algebra • III.D.3. Determine indirect measurements of geometric figures using a variety of methods.
8.8	***Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to:***	
8.8D	**Use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.** ***Supporting Standard***	Use INFORMAL ARGUMENTS TO ESTABLISH FACTS ABOUT THE ANGLE SUM AND EXTERIOR ANGLE OF TRIANGLES, THE ANGLES CREATED WHEN PARALLEL LINES ARE CUT BY A TRANSVERSAL, AND THE ANGLE-ANGLE CRITERION FOR SIMILARITY OF TRIANGLES Including, but not limited to: • Angle – two rays with a common end point (the vertex) • Degree – the measure of an angle where each degree represents $\frac{1}{360}$ of a circle ◦ Unit measure labels as “degrees” or with symbol for degrees (°) • Adjacent angles – two non-overlapping angles that share a common vertex and exactly one ray • Complementary angles – two angles whose degree measures have a sum of 90° • Supplementary angles – two angles whose degree measures have a sum of 180° • Triangle – a polygon with three sides and three vertices ◦ Interior angles of a triangle – angles that are inside of a triangle, formed by two sides of the triangle ◦ Exterior angles of a triangle – angles that are outside of a triangle between one side of a triangle and the extension of the adjacent side

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TEKS# / SE#	TEKS	SPECIFICITY
		◦ Informal arguments to establish facts about triangles • The sum of the measures of the interior angles of a triangle equals 180º. • Adjacent interior and exterior angles create a supplementary pair of angles (the sum of the measures equals 180º). • An exterior angle is equal to the sum of the two non-adjacent interior angles or the remote interior angles. • The sum of the measures of the exterior angles, one at each vertex, of a triangle equals 360°. ◦ Equations to represent the relationships between interior and/or exterior angles and to determine a missing angle measure • Congruent angles – angles whose angle measurements are equal ◦ Arc(s) on angles are usually used to indicate congruency (one set of congruent angles would have 1 arc, another set of congruent angles would have 2 arcs; etc.). ◦ Arcs and tick marks on angles can be used to indicate congruency (one set of congruent angles would have 1 arc with 1 tick mark, another set of congruent angles would have 1 arc with 2 tick marks; etc.). • Vertical angles – a pair of non-adjacent, non-overlapping angles formed by two intersecting lines creating angles that are opposite and congruent to each other • Parallel lines – lines that lie in the same plane, never intersect, and are always the same distance apart ◦ Various orientations including vertical, horizontal, diagonal, and parallel lines of even, uneven, or off-set lengths ◦ Lines that are parallel may or may not contain parallel markings. • Transversal – a line that intersects two or more lines • Alternate interior angles ◦ When two parallel lines are cut by a transversal, congruent alternate interior angles are formed on opposite sides of the transversal and inside the parallel lines. • Same side interior angles ◦ When two parallel lines are cut by a transversal, supplementary same side interior

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		angles are formed between the parallel lines on the same side of the transversal.
		• Alternate exterior angles
		  ◦ When two parallel lines are cut by a transversal, congruent alternate exterior angles are formed on opposite sides of the transversal and outside the parallel lines.
		• Same side exterior angles
		  ◦ When two parallel lines are cut by a transversal, supplementary same side exterior angles are formed outside the parallel lines on the same side of the transversal.
		• Corresponding angles
		  ◦ When two parallel lines are cut by a transversal, corresponding angles (one interior angle and one exterior angle) are formed on the same side of the transversal and on the same side of the parallel lines.
		• Informal arguments to establish facts about the angles created when parallel lines are cut by a transversal
		• Angle-angle criterion for triangles – if two angles in one triangle are congruent to two angles in another triangle, then the measures of the third angle in both triangles are congruent
		• Informal arguments to establish facts about the angle-angle criterion for similarity of triangles
		  ◦ The sum of the measures of the interior angles of a triangle is 180º. If two angles of one triangle are congruent to two angles of another triangle, then the measures of the third angles of the triangles must also be congruent, meaning the two triangles are similar. Therefore, if two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
		
		Note(s):
		
		• Grade Level(s):
		  ◦ Grade 7 wrote and solved equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.
		  ◦ Various mathematical process standards will be applied to this student expectation as appropriate.
		• TxRCFP:

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TEKS# / SE#	TEKS	SPECIFICITY
		◦ Using expressions and equations to describe relationships, including the Pythagorean Theorem • TxCCRS: ◦ III.A. Geometric and Spatial Reasoning – Figures and their properties • III.A.2. Form and validate conjectures about one-, two-, and three-dimensional figures and their properties. ◦ III.C. Geometric and Spatial Reasoning – Connections between geometry and other mathematical content strands • III.C.1. Make connections between geometry and algebraic equations. ◦ III.D. Geometric and Spatial Reasoning – Measurements involving geometry and algebra • III.D.3. Determine indirect measurements of geometric figures using a variety of methods. ◦ VIII.C. Communication and Representation – Presentation and representation of mathematical work • VIII.C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

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ELPS#	SUBSECTION C: CROSS-CURRICULAR SECOND LANGUAGE ACQUISITION ESSENTIAL KNOWLEDGE AND SKILLS.

The English Language Proficiency Standards (ELPS), as required by 19 Texas Administrative Code, Chapter 74, Subchapter A, §74.4, outline English language proficiency level descriptors and student expectations for English language learners (ELLs). School districts are required to implement ELPS as an integral part of each subject in the required curriculum.

School districts shall provide instruction in the knowledge and skills of the foundation and enrichment curriculum in a manner that is linguistically accommodated commensurate with the student's levels of English language proficiency to ensure that the student learns the knowledge and skills in the required curriculum.

School districts shall provide content-based instruction including the cross-curricular second language acquisition essential knowledge and skills in subsection (c) of the ELPS in a manner that is linguistically accommodated to help the student acquire English language proficiency.
http://ritter.tea.state.tx.us/rules/tac/chapter074/ch074a.html#74.4

Choose appropriate ELPS to support instruction.

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